

• CONSERVATION OF MOMENTUM (cont.)

$$\Sigma \vec{F}_{\text{ext}} = \frac{d}{dt} (m_{cv} \vec{v}_{cv}) + \sum_{cs} \dot{m}_o \vec{v}_o - \sum_{cs} \dot{m}_i \vec{v}_i$$

(case 1: Fixed C.V.) $\begin{cases} \vec{v}_{cv} = 0 \\ \frac{d}{dt} (m_{cv} \vec{v}_{cv}) = 0 ! \end{cases}$

$$\begin{cases} \Sigma F_{\text{ext}(z)} = p_e A_e - W + F_b \\ \dot{m}_o v_{o(z)} = -\dot{m} v = -\rho Q v = -\rho A v^2 \\ \quad \quad \quad [\dot{m} = \rho Q] \quad [Q = v A] \end{cases}$$

$$-W + F_b = -\rho A v^2$$

$$F_b = W - \rho A v^2$$

If $\rho A v^2 > W \rightarrow F_b < 0$
 \rightarrow THRUST "FORCE"
 (downward)

FORCE DIAG.
 [F.D.]



MOMENTUM D.
 [M.D.]



\Rightarrow FORCE of Rocket
 on beam \uparrow
 (upward)

The diagram shows a curved pipe with a control volume (C.V.) defined by a dashed red line. The pipe is labeled with $(M.D.)$ and $(F.D.)$. A coordinate system (x, y) is shown. The flow enters at $\dot{m}_i \vec{v}_i$ and exits at $\dot{m}_o \vec{v}_o$. The angle of the pipe is 60° . The forces acting on the control volume are F_b (buoyancy) and $F_{ext}(y)$ and $F_{ext}(x)$ (external forces). The flow is assumed to be inviscid and gravity is neglected.

Neglect: Viscous, gravity

BERNOULLI:

$$\frac{P_i}{\rho} + z_i + \frac{v_i^2}{2g} = \frac{P_o}{\rho} + z_o + \frac{v_o^2}{2g}$$

$(P_i = P_o = 0)$
 $(z_i \approx z_o)$

$\rightarrow V_i = V_o = V$

CONSERVATION of MASS

$$0 = \frac{dm_{cv}}{dt} + \dot{m}_o - \dot{m}_i \rightarrow \dot{m}_i = \dot{m}_o = \dot{m}$$

0 (no accum.)

(conc 1)

$$\frac{d}{dt}(m_{cv} \vec{v}_{cv}) = 0 !$$

$$\sum \vec{F}_{\text{ext}} = \dot{m}_0 \vec{V}_0 - \dot{m}_i \vec{V}_i$$

x-dir $F_{\text{ext}(x)} = \dot{m}_0 V_{0(x)} - \dot{m}_i V_{i(x)} = \dot{m} V \cdot \cos 60^\circ - \dot{m} V$

$(\dot{m} = \rho Q = \rho A V)$ $= \boxed{\rho A V^2 (\cos 60^\circ - 1)}$

y-dir $F_{\text{ext}(y)} = \dot{m}_0 V_{0(y)} - \dot{m}_i V_{i(y)} = -\dot{m} V \sin 60^\circ = \boxed{-\rho A V^2 \sin 60^\circ}$

(Note: A red arrow points down from the boxed result, and a red double-headed arrow indicates the opposite direction of the x-component force.)

FORCE of water on vane } - same magnitude.
 (action-reaction) } - opposite direction.

M.D.

F.D.

β

$\dot{m}_i \vec{v}_i$

$\dot{m}_o \vec{v}_o$

F_b

y

x

(case 1)

- Bernoulli: $v_o = v_i = v$
- Continuity: $\dot{m}_i = \dot{m}_o = \dot{m}$

x-dir

$$F_x = \dot{m}_o v_{o(x)} - \dot{m}_i v_{i(x)} = -\dot{m} v \cos \beta - \dot{m} v = \boxed{-\rho A v^2 (1 + \cos \beta)}$$

$Q: \begin{cases} \beta = 180^\circ \\ \beta = 90^\circ \\ \beta = 0^\circ \end{cases}$

$(\dot{m} = \rho Q = \rho A v)$

• ACCELERATING ROCKET

$$\Sigma F_z = \frac{d}{dt}(m_r v_r) + \dot{m} v_{0(e)}$$

$$\left\{ \begin{aligned} \Sigma F_z &= p_e A_e - W - D \\ \dot{m} v_{0(e)} &= \dot{m} (v_r - v_e) \end{aligned} \right.$$

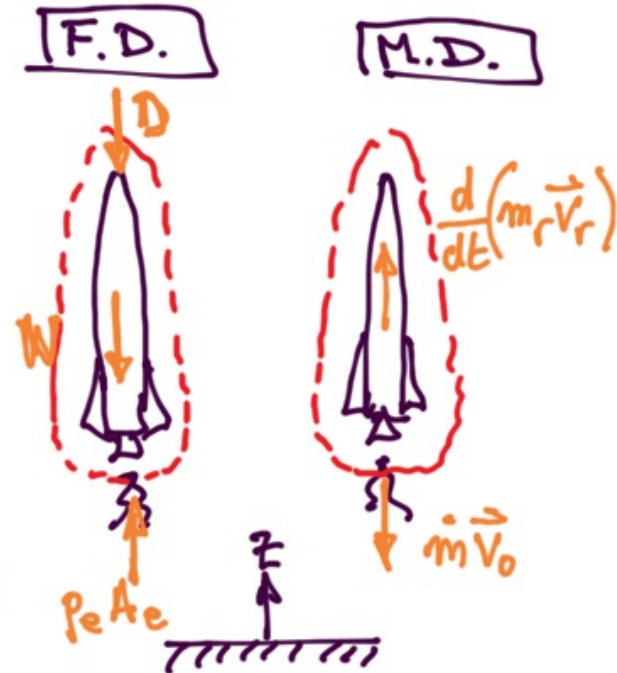
$$\dot{m} v_{0(e)} = \dot{m} (v_r - v_e)$$

$$\left\{ \begin{aligned} \frac{d}{dt}(m_r v_r) &= m_r \frac{dv_r}{dt} + \frac{dm_r}{dt} \cdot v_r \end{aligned} \right.$$

$$\left\{ \begin{aligned} p_e A_e - W - D &= m_r \frac{dv_r}{dt} + \frac{dm_r}{dt} v_r + \dot{m} v_r - \dot{m} v_e \end{aligned} \right.$$

$$\left(\frac{dm_r}{dt} + \dot{m} \right) \cdot v_r$$

0 (continuity)



• THRUST "FORCE" : $T = \dot{m} V_e + p_e A_e$

$$T \gg W, D$$

$$\left. \begin{aligned} T &\simeq m_r \frac{dv_r}{dt} \\ m_r &= m_i - \dot{m} \cdot t \end{aligned} \right\}$$

$$T = (m_i - \dot{m} t) \frac{dv_r}{dt}$$

$$\int_0^{v_r} dv_r = \int_0^t \frac{T}{(m_i - \dot{m} t)} dt$$

$$v_r(t) = \frac{T}{\dot{m}} \ln \left(\frac{m_i}{m_i - \dot{m} t} \right)$$