• CONSERVATION OF MOMENTUM (cont.)

$$\boxed{\sum \vec{F}_{ext} = \frac{d}{dt}\left(m_{cv}\vec{v}_{cv}\right) + \sum_{cs} \dot{m}_o \vec{v}_o - \sum_{cs} \dot{m}_i \vec{v}_i}$$

(case 1 : Fixed C.V.) $\begin{cases} \vec{v}_{cv} = 0 \\ \frac{d}{dt}\left(m_{cv}\vec{v}_{cv}\right) = 0 \ ! \end{cases}$

$$\begin{cases} \sum F_{ext(z)} = \cancel{p_e}^{\,0} A_e - W + F_b \\ \dot{m}_o v_{o(z)} = -\dot{m} v = -\rho Q v = -\rho A v^2 \\ \qquad [\dot{m} = \rho Q] \quad [Q = vA] \end{cases}$$

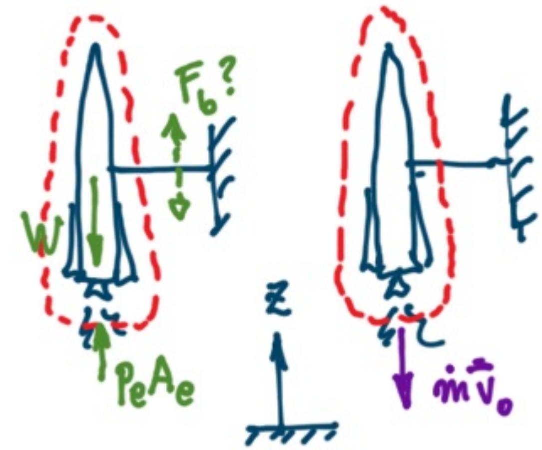
$\rightarrow -W + F_b = -\rho A v^2$

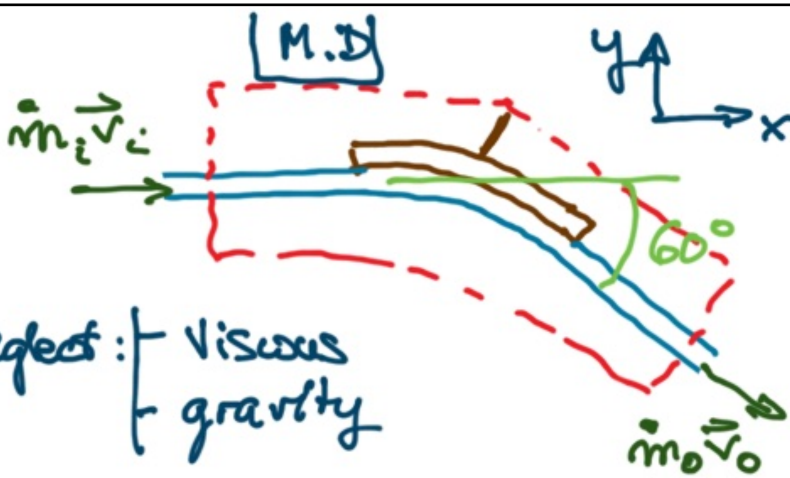
$$\boxed{F_b = W - \rho A v^2}$$

If $\rho A v^2 > W \longrightarrow F_b < 0$ ↓ (downward) $\Rightarrow$ FORCE of Rocket on Beam ↑ (upward)

THRUST "FORCE"

FORCE DIAG. [F.D.] MOMENTUM D. [M.D.]





Neglect: - Viscous
- gravity

(case 1)

$$\frac{d}{dt}\left(m_{cv} \vec{v}_{cv}\right) = 0 \; !$$

• BERNOULLI:

$$\cancel{\frac{P_i}{\gamma}} + z_i + \frac{v_i^2}{2g} = \cancel{\frac{P_o}{\gamma}} + z_o + \frac{v_o^2}{2g}$$

$(P_i = P_o = 0)$

$(z_i \simeq z_o)$

$$\longrightarrow \boxed{V_i = V_o = V}$$

• CONSERVATION of MASS

$$0 = \underbrace{\cancel{\frac{dm_{cv}}{dt}}}_{0 \text{ (no accumul.)}} + \dot{m}_o - \dot{m}_i \longrightarrow \boxed{\dot{m}_i = \dot{m}_o = \dot{m}}$$

$$\Sigma \vec{F}_{ext} = \dot{m}_o \vec{v}_o - \dot{m}_i \vec{v}_i$$

$\boxed{x\text{-dir}}$

$$F_{ext(x)} = \dot{m}_o v_{o(x)} - \dot{m}_i v_{i(x)} = \dot{m}\, v \cdot \cos 60° - \dot{m}\, v = \boxed{\rho A v^2 (\cos 60° - 1)}$$

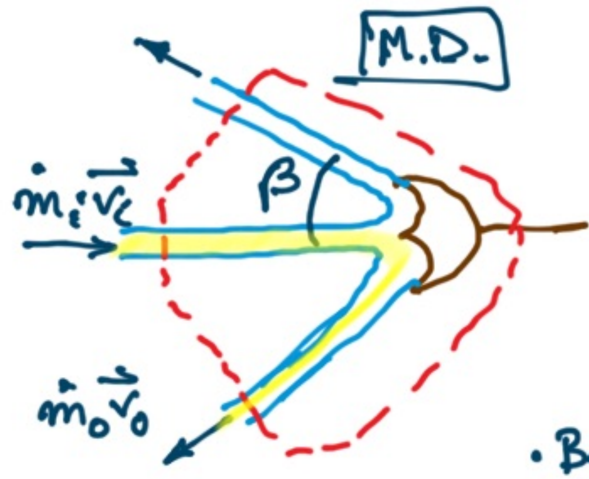
$(\dot{m} = \rho Q = \rho A v^2)$

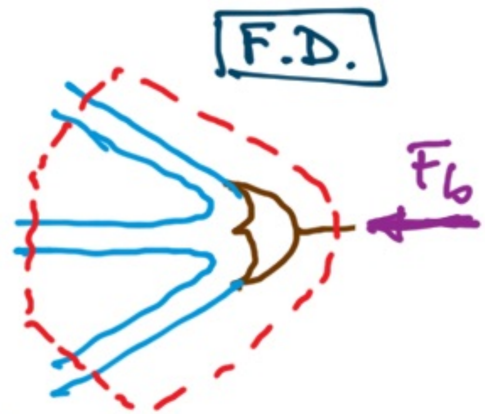
$\boxed{y\text{-dir}}$

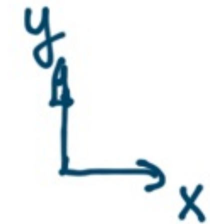
$$F_{ext(y)} = \dot{m}_o v_{o(y)} - \dot{m}_i \cancel{v_{i(y)}}^{\,0} = -\dot{m}\, v \sin 60° = \boxed{-\rho A v^2 \sin 60°}$$

FORCE of water on vane (action-reaction)

- same magnitude.
- opposite direction.







(case 1)

- Bernoulli: $v_o = v_i = v$
- Continuity: $\dot{m}_i = \dot{m}_o = \dot{m}$

x-dir

$$F_x = \dot{m}_o v_{o(x)} - \dot{m}_i v_{i(x)} = -\dot{m} v \cos\beta - \dot{m} v = \boxed{-\rho A v^2 (1+\cos\beta)}$$

$$(\dot{m} = \rho Q = \rho A v)$$

$$Q: \begin{cases} \beta = 180^\circ \\ \beta = 90^\circ \\ \beta = 0^\circ \end{cases}$$

• ACCELERATING ROCKET

$$\Sigma F_z = \frac{d}{dt}(m_r v_r) + \dot{m}\, v_{o(z)}$$

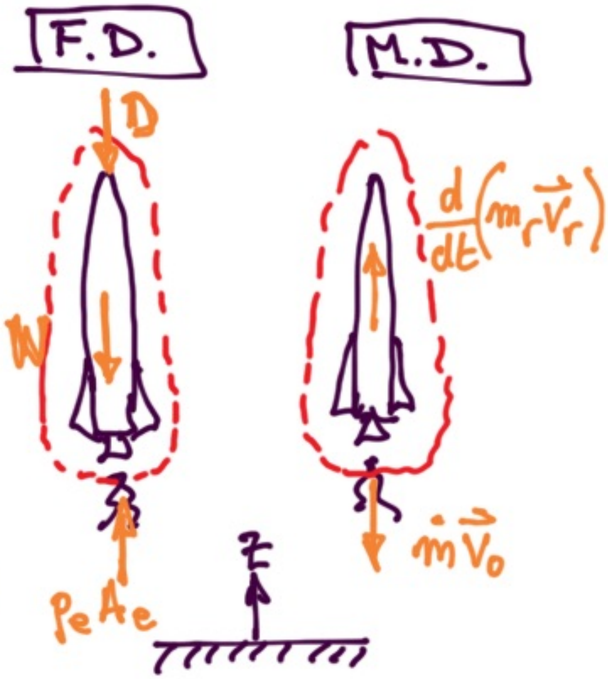
$$\Sigma F_z = p_e A_e - W - D$$

$$\dot{m}\, v_{o(z)} = \dot{m}\,(v_r - \mathbb{V}_e)$$

$$\frac{d}{dt}(m_r v_r) = m_r \frac{dv_r}{dt} + \frac{dm_r}{dt}\cdot v_r$$

$$p_e A_e - W - D = m_r \frac{dv_r}{dt} + \underbrace{\frac{dm_r}{dt} v_r + \dot{m}\, v_r}_{\left(\frac{dm_r}{dt} + \dot{m}\right)\cdot v_r} - \dot{m}\,\mathbb{V}_e$$

$$\left(\cancel{\frac{dm_r}{dt} + \dot{m}}\right)\cdot v_r \rightarrow 0 \text{ (continuity)}$$



• THRUST "FORCE" : $T = \dot{m} V_e + p_e A_e$

$T \gg W, D$

$$\left.\begin{aligned} T &\simeq m_r \frac{dv_r}{dt} \\ m_r &= m_i - \dot{m} \cdot t \end{aligned}\right\}$$

$$T = (m_i - \dot{m} t) \frac{dV_r}{dt}$$

$$\int_0^{V_r} dV_r = \int_0^t \frac{T}{(m_i - \dot{m} t)} dt$$

$$\boxed{V_r(t) = \frac{T}{\dot{m}} \ln\left(\frac{m_i}{m_i - \dot{m} t}\right)}$$